Teacher Knowledge for Maths Aotearoa Book 1A and Wilkie Way

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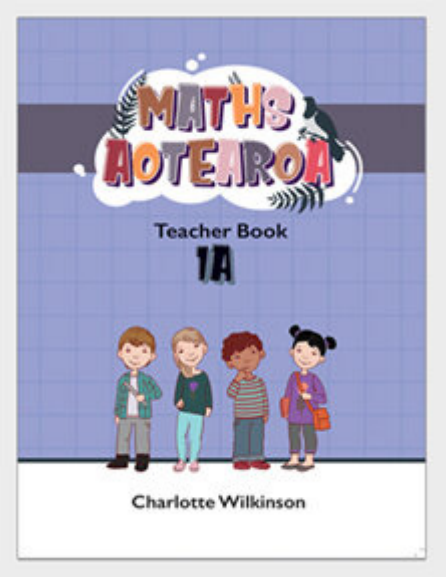
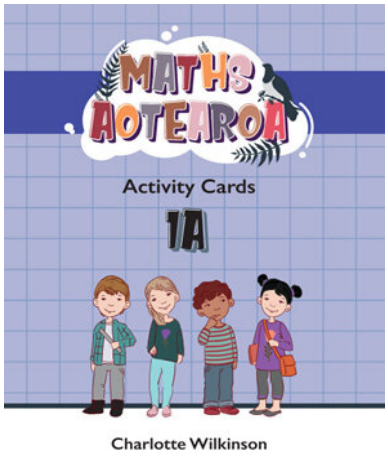




Wilkie Way is trademarked to NCWilkinsons Ltd and Charlotte continues to develop copyrighted resources for primary mathematics wherever a need is identified, including resources to directly support the use of Maths Aotearoa.

Most Wilkie Way resources are available to download from wilkieway.co.nz through an annual subscription. Individual membership allows for personal use within your own class only.
School membership allows all teachers in a school to create an account and resources can be used across the whole school.
Teacher Handbooks for Fractions, Decimals & Percentages, Numbers & The Number System and Arithmetic Operations, ready to use laminated card Dice & Counter games and Assessment Screen booklets are available from the onlne store at wilkieway.co.nz

Maths Aotearoa Book 1A is organised into four large sequenced units.

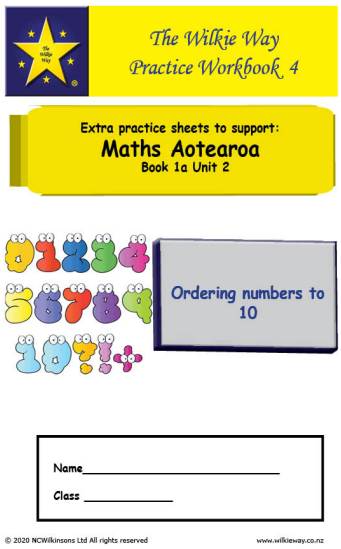
Unit 1: Making Sense of Small Numbers
Unit 2 Exploring Numbers to 10
Unit 3 Combining, Comparing and Ordering
Unit 4 Combining, Grouping and Sharing

Each unit within Maths Aotearoa Book 1A is made up of 6 elements. The best practice for delivery of content should be through a mixture of explicit teaching, guided practice, flexible grouping and independent activity, including play based activities with interactive dialogue to develop deeper thinking and language development in meaningful contexts. The elements within a unit are interconnected so it is important that connections are made to develop a robust foundational understanding of mathematics not just number, or space or measurement or statistics.

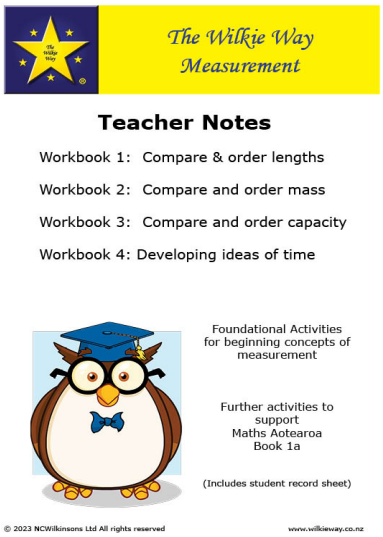
The first two units are the essential foundation blocks for mathematical success. Some students on entry to school will need to start right at the beginning, some will start at the beginning and make very fast progress, some will already have significant knowledge and understanding. Undertaking a baseline assessment is useful and especially so to identify those students with a lot of knowledge and those whose progress requires more support.

A baseline assessment is available to Wilkie Way members under the heading Assessment in the resource directory.

Each unit has suggested classroom activities, many using equipment readily available in a junior classroom. There are also a total of 100 activity cards to support units 1 - 4. Many of the activities are practical and can be used multiple times. Some have activities on both sides of the cards and some have teacher information for further teacher guidance in developing the mathematics from the activity.



Further practice for each unit can be found in 12 printable number and algebra workbooks and 4 measurement workbooks and teacher notes available to Wilkie Way members.



Tracking Progress:

Maths Aotearoa provides tracking sheets to record student progress through out their learning journey.

Progress tracking sheets can also be found under the Assessment heading in the Wilkie Way membership directory. These tracking sheets have been updated to reflect the refreshed curriculum by using the 6 headings of the refreshed curriculum rather than the three headings of the previous curriculum. The mathematics hasn't changed!

Maths Aotearoa Book 1

Unit 1 - Making Sense of Small numbers:

Counting, Sequencing & Patterning	Reading Numerals & Making groups up to 6	Comparing & Ordering numbers to 6
Statistics Sorting & Matching	Working with Shapes Continued through Unit 2	Describing Position Continued through Unit 2

From the outset of a student's journey in learning mathematics teachers must keep in mind the essence of mathematics is in the patterns and relationships.

"What is mathematics and statistics about?

Mathematics is the exploration and use of patterns and relationships in quantities, space and time. Statistics is the exploration and use of patterns and relationships in data."

Seeking patterns and relationships requires students to identify and notice attributes and whether they are the same or different. They will also need to consider if the attributes noticed relate to each other and how they relate.

This noticing occurs pre-number (starting from the moment a child is born) as they consider visual attributes. With increasing knowledge and language they are able to verbalise for example colour and shape. The attributes may have to do with measure – size, mass etc. and a substantial language base is required to support the developing concepts and knowledge.

Many of a student's early experiences are in the geometric strand of the mathematics curriculum through their interactions with the world around them. Making sense of the geometric elements requires development of language to describe, explain and think.

Students develop an understanding of numbers through real experiences.
Humans have counted and compared quantities for a very long time (since the beginnings of civilisations). The ability to recognise the difference in quantity between small numbers (one, two and three) without counting is called subitizing. Much research suggest this ability can be found in very young babies but further research also suggests there is a developmental brain maturation element to this as well. The picture remains complicated and the complexity in developing number understanding requires areas of the brain to work together. Visualisation and symbolic representations of numbers develop interdependently. Variations in experiences, including language, counting practice, and representations seem likely to affect the development.

We have a distinct set of number words in a particular order that need to be mastered. There is a big difference between being able to say the number words in sequence and understanding numbers.
A number is an abstract idea and to be a number it must be a quantity of something. Students need to come to the understanding that the counting words are assigned to an individual item and the last number in the counting set tells you how many there are. To achieve this, a student must learn the names for the numbers and the order in which the words come. They must use a one to one relationship (one word to one item) and learn to recognise the symbol that represents the counting word and the quantity in the group of objects. This is the cardinal aspect of number.

They must also begin to consider the position of individual numbers in a sequence of numbers using language of before, after, between, first, second, third. This is a more abstract idea and is the ordinal aspect of number. It is reliant on an understanding of sequencing which begins pre-number and is essential to reading and writing as well as mathematics. Much of what we do occurs in a sequence of events – for example getting dressed, daily routines.
To assist the development of ordinal number, to focus students on the position of the number word in the sequence, items to be counted should be arranged in a row (or column). When a student counts along a row he/she is not only setting up a one – one correspondence with the set of number names, he/she is also naming each item by its position in the row. Number two is the second in the row, number three is the third. When the student is counting he/she is not necessarily thinking of the set of three items he/she has counted but that the item is the third in the row. For the ordinal aspect of number to be present in the activity, the things counted must be seen in order.

Many young children struggle with sequencing and it certainly has a developmental aspect. Visual sequencing can be slow to develop but for most students just requires lots of opportunities to practice.

Unit 2 Exploring Numbers to 10

Numerals & Making Groups to 10	Compare and Order Numbers to 10	Joining & Partitioning Sets of Objects
Exploring Groupings within 5	Working with Shapes Continued from Unit 1	Describing Position Continued from Unit 1

Students have an idea that numbers are used for counting sets of objects. They extend their numeral identification to all digits 0 – 9 including the first combination of digits, number 10.

Their understanding of numbers expands from the natural numbers (counting numbers) to the whole numbers, which now includes zero. For zero to be a number it must represent nothing of something. If students see zero as just nothing then they will often disregard it as not important.

Number 10 should be identified as a special number from the outset even though students will not yet be able to use a set of 10 as a counting unit with understanding. The decade names should be modelled and used in the classroom,

- Keeping a record of the number of days in school by adding a lollipop stick to a pot and when a group of ten is made move it to the next pot. Keep a number chart to show the number of groups of ones and the number of groups of ten.
- Making reward sticker charts in groups of 10.
- Create opportunities to count in groups of ten where the ten ones are evident.

This work is helping to build vocabulary. There are 29 number names to master in order to count to 100.

Along with language building they are developing their understanding of the cardinal and ordinal aspects of number. Work on positional language continues as in Unit one and students should begin connecting the idea of one more with the number after and one less or fewer with the number before in the counting sequence in ones.
As they learn to quantify, noticing “how many” becomes another attribute for sorting: more, less, equal group.
Grammatically “fewer” can only be used when referring to countable objects – “less” is often used for both countable objects and singular mass nouns (money, love etc.). Students need to have the words fewer and fewest in their vocabulary as they will meet the words during their mathematics learning journey.
Likewise “How much?” is only used for singular mass nouns while “how many?” is only used for countable nouns – a very common grammatical error which should not go uncorrected.

Sorting shapes should now also include the quantifiable aspects of the shapes. For example

- The number of straight sides
- The number of corners
- The number of squares that can be counted on a cube

Partitioning sets begins the whole/part/part structure which underpins the inverse relationship between addition and subtraction. Students need to see that a whole number is made up of smaller whole numbers before they can combine whole numbers to make a bigger whole number. From these experiences they are being exposed to the commutative property of addition.

Special attention is given to groupings with five for students to focus on beginning to combine small numbers to make a bigger number and recognise the patterns within pairs to make the same number. The idea that a number can be made of different combinations helps build conservation of number. The understanding that the count of all objects in a set, in any order will always give the same number and the last number spoken in the sequence tells you how many. These patterns observed in these activities are the beginnings of commutative addition and understanding the next counting number is the number after and is the result of adding one more to the set. The number before is the result of taking one away from the set. These activities also begin the understanding of the inverse relationship between addition and subtraction, if I take one (subtract) from one part and give it (add) it to the other part the total amount does not change.

Units 3 and 4 begin the journey of operations with numbers - addition, subtraction, multiplication and division. Linear measure activities connect to developing concepts of addition and subtraction and the statisitics and probability elements which continue through both units 3 and 4 extend student number knowledge by developing early multiplicative ideas about numbers.

Unit 3: Combining, Comparing and Ordering

Addition & Subtraction	Attribute of Measure	Comparing Measure
Shape & Pattern	Statistics Continued through Unit 4	Probability Continued through Unit 4

With increasing number knowledge, counting can be used to describe shapes more accurately by using number of sides, number of corners and defining features using increasing geometric language. Consideration should be given to the purpose of particular shapes in our everyday lives. This is important to design and technology. Does the shape roll, stack or slide? When describing shapes consider the attributes a shape does not have. Learning to consider the negative attributes can be quite difficult for young students who are reliant on what they can see. However much of mathematics is abstract and students need to learn to image what is and what isn't to develop spatial and numeric reasoning. Student/teacher dialogue should expand to encourage spatial thinking.

Spatial thinking is essential to problem solving. A visual pictorial mode of thought enables a student to visualise a problem geometrically – hence the problem solving strategy of draw a picture.

Tessellating patterns introduces the idea of covering a surface with no gaps or overlaps which will eventually lead to measurement of area.

Linear measure underpins ideas about addition and subtraction in a very meaningful context. A bar model, using materials such as Cuisenaire or number strips representing addition and subtraction is joining, separating, comparing and ordering units of length.

The concepts of measure are the same for whatever is being measured. Students need to consider what can be measured: Length, mass, capacity, volume, area, temperature, angle, time. There is an enormous amount of vocabulary associated with concepts of measure that students need to develop as their ideas about measurement evolve. At this level students are identifying the attributes that can be measured and then making comparisons in practical situations with appropriate dialogue focusing on developing the associated vocabulary. Direct comparison as used in measurement is also the basis for additive comparison or finding the difference in quantity.

Time and in particular the passage of time is a difficult concept for children to judge. It should be taught in association with the specific times relevant to the students' lives. Again there is an extensive amount of vocabulary associated with time concepts. Learning in this area is very much based on experience and use must be made of every learning opportunity that arises during the school day to support understanding of time.

The statistical display of a uniform pictograph with objects allows for the direct comparison of quantities, including a set of zero in a meaningful situation. Direct comparison statements need to be asked of the data like:

- How many?
- Which set has more than another set?
- Which set has fewer/less than another set?

- Which set has the most?
- Which set has the least/fewest?

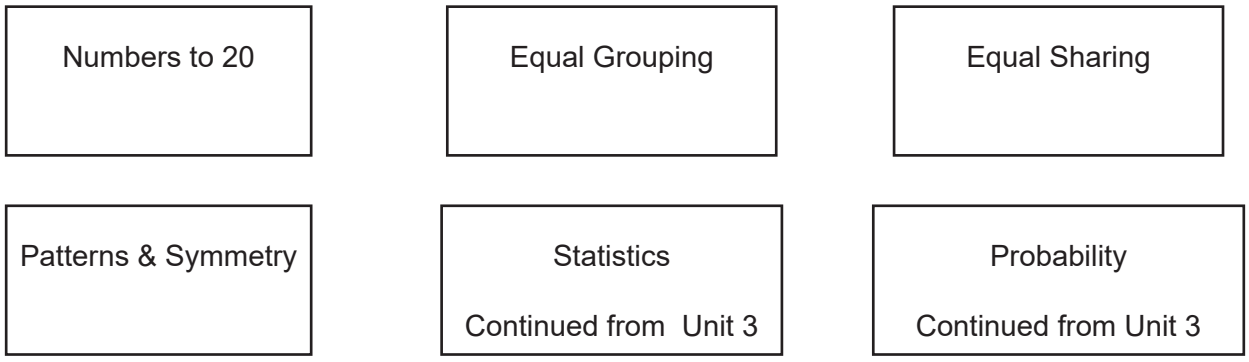
From a uniform pictograph students should be able to answer the questions without counting the objects but by making a measurement comparison. Focusing on the comparison focuses the thinking on the difference between quantities and introduces questions like how many more and how many less.

Introduce vocabulary of chance. Students need to realise that while there are many ways of partitioning a number, there is a finite number of pairs to make each number. These are known as the basic facts. While learning basic facts is very important it should not become the only focus of primary mathematics and used as a measure for mathematical success.

Addition and subtraction concepts continue to develop as number knowledge increases and students should be beginning to develop a solid understanding of the cardinality of numbers (number conservation). Students who have not attained the cardinal principal will be delayed in their ability to add and subtract with meaning. It would be hard to move them from counting all the objects when left to their own devises although they can learn the “trick” of counting on as a procedure under teacher guidance and if they know the number after the given number.

With some conceptual understanding of addition and subtraction it is appropriate to introduce the symbols representing the concepts by modelling their use in meaningful contexts. A meaningful concept requires the use of a story problem. Initially these problems will be verbal but using simple words and picture clues, students need to be taught how to approach the reading of mathematics.
Reading mathematics from the outset requires comprehension of the problem. Students should be encouraged to draw a simple picture of the problem before focusing on the symbolic equation for the situation. If students are only encouraged to write the equation for the problem they will only focus on the numbers and key words to give a clue to whether the problem is an addition or a subtraction. While at this stage the problems are straight forward, if students have not learnt how to read mathematics their problem solving later in their learning journey will be severely compromised.

Unit 4: Combining, Grouping and Sharing



Some students will already know the counting sequence to 20 and beyond but this unit is looking more closely at the teen words and the ty words. For many students the similarity in sound of these words leads to a teen/ty confusion. Connecting the word, symbol and representation of the quantity in a student’s mind is essential. This will only occur through multiple experiences of seeing the difference in material representation of teen numbers and ty numbers. Using Te Reo Maori emphasises the tens and ones structure of the teens numbers and the number of groups of ten for ty number.

A series of lessons exploring teens and tys is available under the Place Value heading in the Wilkie Way membership directory. Also resources for activities for understanding the teen and ty number symbols. The beginnings of place value are understanding the value of the digit in its position (place) in a number.

An understanding of quantity provides the foundation for understanding multiplication. A quantity is a characteristic of objects that can be counted or measured and consists of a number and a unit. The number words describe the number portion of a quantity and can be shown with representations, for example five counters. The number on its own does not exist. It can only be described in relation to something, e.g the counters. A count is a discrete quantity that answers the question “How many?”

Counting has begun with counting in ones and understanding the count is the cardinal principle of number. Students who have achieved or are close to achieving cardinality of number and have gained the knowledge of the number after in a sequence will be beginning to count on to solve addition type problems or use recall of known addition facts. Counting backwards to solve subtraction problems requires not only the knowledge of the number before but also a firm knowledge of the backward counting sequence. To use recall of subtraction facts students need to have knowledge of and apply the inverse relationship between addition and subtraction.

Understanding counting now progresses to counting in equal size groups larger than one.
Learning to rote count in twos, fives and tens does not mean students have an understanding of equal grouping. Experiences of counting objects in groups other than one is required. As students use subitizing small groups to make a count of items around them, they acquire the skill of counting in twos with an understanding that items number one, three, five etc are still present in the count.

Developing the counting sequences in twos, threes, fives and tens is necessary before students can use these sequences to solve equal grouping type problems.
From the experiences of counting items in equal groups, of adding equal groups the ideas of multiplication begin to develop. The first multiplication facts a student recalls will be the doubles but these are recorded at this stage as addition facts.

A very clear understanding of equal grouping and learning to use an equal group as a counting unit is essential to working with the equal group of ten. Work on place value, considering the value of the digit in the place it occupies in a number requires the use of a group of ten as a counting unit that behaves in the same way as groups of one but in a different place (column) in any number.

Division is the inverse operation of multiplication. Young students first use the idea of division at a very early stage, well before the concept of multiplication has been developed to any degree. A student uses two fundamentally different types of division. One is the act of equal sharing. While this sharing or partitioning type of division is mathematically more complex, students often use it earlier as no counting is needed to share out a non-counted set of items (bag of lollies) between an uncounted group of children. The sharer just has to keep going around the group giving one at a time until no lollies are left. It does not become numerical until specific numbers are given For example: Share out a packet of ten cookies between a group of five students.

The second type is when he/he works out how many students could have two cookies each from a packet of ten cookies. This aspect of division, grouping or quotition is mathematically simpler, but seems at first to young students to be completely unrelated to the equal sharing.

Early experiences of division, sharing and grouping must be set in practical situations so that an appropriate method for solving the problems can be used. Use equal sharing for sharing problems, and equal grouping for grouping problems.

While students have experienced the idea of small numbers embedded in larger numbers they are introduced to the idea of fractions. At this level the vocabulary of half, halves and quarters rather than the symbols is of greater importance and understanding the equality of two equal parts for halves or four equal parts for quarters. This extends their understanding of partitioning into sharing an object or a set of objects into equal parts.
Partitioning involves sharing a number of objects into equal sized groups (a discrete context) or partitioning an object or shape into the same size pieces (a continuous context)
The continuous context of partitioning underpins the measurement concepts of equal size units, with no gaps or overlaps.

Making two halves the same gives rise to the notion of a balance or equality. Balancing leads to finding shapes that looked balanced. Making shapes that look balanced is a fascinating game with plenty of variety yet it leads to one of the most important concepts of mathematics, the idea of symmetry. The symmetry created by reflection is a valuable tool for discovering properties of shapes.

Folding activities are a foundational activity to exploring the idea of angles at a later stage.

As you have read through this document you begin to realise the enormous amount of learning a student needs to do in their first year at school to set firm foundations for further mathematics learning,
This learning will not happen by chance

Extract from ERO document: Making it Count: Teaching Maths in Year 1 - 3
(https://evidence.ero.govt.nz/documents/making-it-count-teaching-maths-in-years-1-to-3)

Teachers are not as well set up as they could be for the deliberate, structured approach to maths teaching that the evidence tells us makes the difference.

Maths in early primary school matters.Maths achievement in the primary years is linked to later success across a range of life outcomes, like higher education achievement, better jobs, better income, and social mobility. Maths results have even been shown to impact on national economies.

In the early years of primary school, teachers have the opportunity to set the scene for their young maths students through purposeful strategies and explicit instruction. It is in these early years that students learn about the building blocksof all future maths learning, and develop their understanding of how capable they are as maths students. Maths learning builds on itself and gets more complex over a student's time in school, so getting the foundation right is really important. Any misunderstandings, shortcuts, poor self-belief, or lack of engagement in these early years sets a poor foundation for years to come.
To make this happen, teachers need to ensure that all students benefit from high-quality maths experiences every day. When maths is a consistent and engaging feature of the daily classroom programme, students have lots of opportunities to make connections, cement new learning, think and talk in maths terms, and explore maths ideas. Purposeful and evidence-based maths practices are the key to more confident maths students in the future.

There are two enablers for great maths teaching practice

These are the enablers that need to be in place before teachers can do their best maths teaching.

→Enabler 1: Teacher knowledge
Teachers need to be confident in their own maths knowledge and skills, to be ready to teach them to students. They also need to understand what works best for young students: the specific teaching strategies that are most effective in setting students up for this crucial time in their maths journey. This includes being clear about how to structure their teaching to develop important maths understandings over time, while avoiding misunderstandings or shortcuts which negatively impact on later learning.

→Enabler 2: School culture and a whole school curriculum
Teachers' school settings can promote good maths practice through a clear, shared understanding of quality maths teaching. This involves clearly setting out what maths teaching and maths progress looks like in a documented, structured whole school curriculum, and by supporting teacher understanding with great learning and collaboration opportunities. It's useful when schools have an embedded culture of being open to learning, sharing, and continually improving.